Riding with the four horsemen and the multivariate normal tempered stable model

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XVI Workshop on Quantitative Finance
January, 30 - Parma

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Outline

- The four horsemen
- Literature review
- Tempered stable (TS) and tempered infinitely divisible (TID) distributions and processes
- The multivariate normal tempered stable model (3 horsemen)
- The multivariate normal tempered stable model with volatility clustering (4 horsemen)
- Expectation-maximization maximum likelihood parameter estimation
- Market data calibration
- A risk assessment study
The four horsemen

If you google "the four horsemen":

... but here we want to study some stylized facts about multivariate financial time series of equity (index) returns:

- heavy tails
- negative skew
- asymmetric dependence
- volatility clustering

Allen and Satchell (wp, 2014) refer to these four stylized facts as the **four horsemen**.
Introduction

Literature review - multivariate models

- Elliptical and generalized elliptical heavy tail distributions (Frahm, 2004, Kring et al., Dominicy et al. 2013)
- Multivariate normal tempered stable - MNTS (Kim et al., 2012)
- Multivariate modelling with jumps (Leoni and Schoutens, 2008, Guillaume, 2013, Tassinari and Bianchi, 2014)
- Expectation-maximization (EM) maximum likelihood estimation (Liu and Rubin, 1994)
- Discrete-time multivariate Garch model with non-normal innovation (Paolella and Polak, 2013)
- Fast Fourier transform algorithm to approximate the density / distribution function (Stoyanov and Racheva-Iotova, 2004, Scherrer et al., 2012, Bianchi et al., to appear, Ballotta and Kyriakou, to appear)
Introduction

Literature review - tempered stable models

- The *Tempering stable processes* paper appears (Rosiński, 2002-2007)
- Tempered stable OU processes extensively studied by Zhang and Zhang (2008-2009), Kawai et al. (2011-2012), Bianchi et al. (to appear)
- The modified tempered stable (MTS) distribution (Kim et al., 2008) is studied, but it is not in the TS class
- By following the Rosiński approach TS distributions and processes are further extended: tempered infinitely divisible (TID) (Bianchi et al., 2010), generalized TS (Rosiński and Sinclair, 2010), and p-TS (2012, Grabchak) classes are introduced.
- The introduction of TID distributions and processes (Bianchi et al., 2010) is motivated by some practical problems in the field of quantitative finance (i.e. GARCH models with heavy-tailed innovations).
- Multivariate normal tempered stable - MNTS (Kim et al., 2012)
- Stock models driven by tempered stable processes (Küchler and Tappe, 2014 and reference therein)
Theorem

A probability law $\mu$ of a real-valued random variable $X$ on $\mathbb{R}$ is infinitely divisible with characteristic exponent $\psi$, 

$$
\int_{\mathbb{R}} e^{iux} \mu(dx) = e^{\psi(u)} \quad \text{for } u \in \mathbb{R}
$$

if and only if there exists a triple $(a_g, \sigma, \nu)$ where $a_g \in \mathbb{R}$, $\sigma \geq 0$, $\nu$ is a measure on $\mathbb{R}\setminus\{0\}$ satisfying

$$
\int_{\mathbb{R}\setminus\{0\}} (1 \wedge x^2) \nu(dx) < \infty
$$

and $g$ is a given truncation function such that

$$
\psi(u) = iau - \frac{1}{2} \sigma^2 u^2 + \int_{\mathbb{R}\setminus\{0\}} (e^{iux} - 1 - iu g(x)) \nu(dx)
$$

for every $\theta \in \mathbb{R}$. 

Lévy-Khintchine formula
Some definitions

Recall that the characteristic function of a real random variable $X$ is defined as

$$\Phi_X(u) = E[e^{iuX}]$$

and given the characteristic function $\Phi_X(u)$ of a law $X$, density and cumulative distribution functions can be derived via the Fourier inversion formula, that is

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iux} \Phi_X(u) du$$  \hspace{1cm} (1)

or

$$F(x + \frac{h}{2}) - F(x - \frac{h}{2}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iux} \Phi_X(u) \frac{2}{u} \sin \frac{hu}{2} du,$$  \hspace{1cm} (2)

where $h > 0$. 
The starting point

The Lévy measure $\nu_0$ of a $\alpha$-stable distribution on $\mathbb{R}^d$ in polar coordinates is of the form

$$\nu_0(dr, du) = r^{\alpha - 1} dr \sigma(du)$$

where $\sigma$ is a finite measure on $S^{d-1}$. Thus, let us consider

$$\nu(dr, du) = r^{\alpha - 1} q(r,u) dr \sigma(du).$$

The tempering function $q$ can be represented as

$$q(r, u) = \int_0^\infty e^{-rs} Q(ds|u) \quad \text{TS case}$$

and

$$q(r, u) = \int_0^\infty e^{-r^2s^2/2} Q(ds|u) \quad \text{TID case} \quad \left( \int_0^\infty e^{-r^p s} Q(ds|u) \quad \text{p-TS case} \right)$$

where $\{Q(\cdot|u)\}_{u \in S^{d-1}}$ is a measurable family of Borel measures on $(0, \infty)$. We analyze only the simplest case $Q(ds|\pm 1) = \delta_{\lambda_{\pm}}(s)ds$. 
The Lévy measure of the $\alpha$-stable distribution is $0 < \alpha < 2$

$$\nu_{S\alpha}(z) = \frac{C}{z^{1+\alpha}}1_{z>0} + \frac{C}{|z|^{1+\alpha}}1_{z<0}.\quad (1)$$

The Lévy measure of the classical tempered stable (CTS) distribution is

$$\nu_{CTS}(z) = q_{CTS}(z)\nu_{S\alpha}(z) = \frac{Ce^{-\lambda_+ z}}{z^{1+\alpha}}1_{z>0} + \frac{Ce^{-\lambda_- |z|}}{|z|^{1+\alpha}}1_{z<0},\quad (2)$$

and of the rapidly decreasing tempered stable (RDTS) distribution is

$$\nu_{RDTS}(z) = q_{RDTS}(z)\nu_{S\alpha}(z) = \frac{Ce^{-\lambda_+ z^2/2}}{z^{1+\alpha}}1_{z>0} + \frac{Ce^{-\lambda_- z^2/2}}{|z|^{1+\alpha}}1_{z<0}.\quad (4)$$

They are similar (but different!). In the limiting case $\alpha \to 0$, equation (3) becomes the Lévy measure of the VG law.
CTS vs RDTS

CTS with parameters $C = 1$, $\lambda_+ = 1$, and $\alpha = 0.9$ ($\Delta t = 1$)

RDTS with parameters $C = 1$, $\lambda_+ = \sqrt{2}$, and $\alpha = 0.9$ ($\Delta t = 1$)
Characteristic functions

[CTS] \[ \phi_{CTS}(u) = E[\exp(iuX)] = \exp(iu(m - C\Gamma(1 - \alpha)(\lambda_+^{\alpha-1} - \lambda_-^{\alpha-1})) + C\Gamma(-\alpha)((\lambda_+ - iu)^{\alpha} - \lambda_+^{\alpha} + (\lambda_- + iu)^{\alpha} - \lambda_-^{\alpha})) \]

[VG] \[ \phi_{VG}(u) = \exp(iu(m - C(\lambda_+ - \lambda_-)\lambda_+^{\alpha-1}\lambda_-^{\alpha-1})) - C \log(\lambda_+\lambda_- + (\lambda_+ - \lambda_-)iu + u^2) + C \log(\lambda_+\lambda_-) \]

[RDT≡] \[ \phi_{RDTS}(u) = \exp(ium + CG(iu; \alpha, \lambda_+) + CG(-iu; \alpha, \lambda_-) \]

\[ G(x; \alpha, \lambda) = 2^{-\alpha/2-1} \lambda^{\alpha} \left( \Gamma\left(-\frac{\alpha}{2}\right) M\left(-\frac{\alpha}{2}, \frac{1}{2}; \frac{\sqrt{2x}}{2\lambda}\right)^2 \right) + \frac{\sqrt{2x}}{\lambda} \Gamma\left(\frac{1 - \alpha}{2}\right) M\left(\frac{1}{2} - \frac{\alpha}{2}, \frac{3}{2}; \frac{\sqrt{2x}}{2\lambda}\right)^2 \]

\[ - \frac{\sqrt{2x}}{\lambda} \Gamma\left(\frac{1}{2} - \frac{\alpha}{2}\right) - \Gamma\left(-\frac{\alpha}{2}\right) \]
MNTS model: definitions

Let $\alpha$, $\lambda$, and $C$ be positive constants, and $0 < \alpha < 2$. The process $S_t$ is said to be a CTS subordinator with parameters $(\alpha, \lambda, C)$ if $0 < \alpha < 1$ and the characteristic function of $S_t$ is given by

$$\phi_{S_t}(u) = E[\exp(iuS_t)] = \exp \left( tC\Gamma(-\alpha)((\lambda - iu)^\alpha - \lambda^\alpha) \right), \quad (6)$$

the Laplace exponent is

$$l_{S_t}(u) = \ln \psi_t \left( \frac{u}{i} \right) = tC\Gamma(-\alpha)((\lambda - u)^\alpha - \lambda^\alpha) \quad (7)$$

and its mean and variance are

$$E [S_t] = -\alpha tC\Gamma(-\alpha)\lambda^{\alpha-1}, \quad (8)$$
$$VAR [S_t] = \alpha(\alpha - 1) tC\Gamma(-\alpha)\lambda^{\alpha-2}. \quad (9)$$
Definitions

Let $Y_t$ be a multivariate process defined as

$$Y_t = \mu t + \theta S_t + \sqrt{S_t}D\sigma AZ,$$

where $Z$ is a multivariate normal distribution with zero mean and unit variance-covariance matrix $I$ of dimension $n$.

- $S_t$ is the subordinator with Laplace exponent $l_{S_t}(s)$ and density function $h$ defined in equation (6) with $\alpha = a/2$.

- $A$ is the lower Cholesky decomposition of a correlation matrix $\Omega$, that is, $\Omega^{1/2} = A$; furthermore, $\Sigma^{1/2} = D_\sigma A$, where $D_\sigma$ is a diagonal matrix with diagonal entries $\sigma \in \mathbb{R}^n$.

- $\mu$ and $\theta$ are vector in $\mathbb{R}^n$.

Then the process $Y_t$ is referred to as the multivariate normal tempered stable (MNTS) process with parameters $(a, \lambda, C, \theta, \mu, \Sigma)$. 
The characteristic function of $Y_t$ defined in equation (10) is given by

$$
\Psi_{Y_t}(u) = \exp \left( itu' \mu + tl_{S_1}(g(u)) \right).
$$

(11)

where $l_{S_1}(\cdot)$ is the Laplace exponent of the subordinator $S_t$ at time $t = 1$, and $g(u)$ is the characteristic exponent of the multivariate Brownian motion, that is

$$
g(u) = iu' \theta - \frac{1}{2} u' \Sigma u
$$

(12)

with $u \in \mathbb{R}^n$, and where the matrix $\Sigma$ has elements $\Sigma_{jk} = \sigma_j \sigma_k \rho_{jk}$. Since $\Sigma$ is a variance-covariance matrix, we can rewrite equation (12) in the following form

$$
g(u) = iu' \theta - \frac{1}{2} u' \Sigma \Omega \Sigma u,
$$

(13)

where $D_\sigma$ is a diagonal matrix with diagonal $\sigma \in \mathbb{R}^n$, and $\Omega$ is the correlation matrix of the Brownian motions with elements $\rho_{jk}$. 
The MNTS model

Properties

Thus, equation (11) can be written as

$$\Psi_{Y_t}(u) = \exp \left\{ t \left[ iu' \mu + C \Gamma \left( -\frac{a}{2} \right) \left( \left( \lambda - iu' \theta + \frac{1}{2} u' \Sigma u \right)^{\frac{a}{2}} - \lambda^{\frac{a}{2}} \right) \right] \right\}. \quad (14)$$

Setting $u_i = 0, \forall i \neq j$, into (14) we get the characteristic function of the $j$-th margin

$$\Psi_{Y_t}(u_j) = \exp \left\{ t \left[ iu_j \mu_j + C \Gamma \left( -\frac{a}{2} \right) \left( \left( \lambda - iu_j \theta_j + \frac{1}{2} u_j^2 \sigma_j^2 \right)^{\frac{a}{2}} - \lambda^{\frac{a}{2}} \right) \right] \right\}. \quad (15)$$

Mean and variance have the following form

$$E \left[ Y_t^j \right] = \mu_j t + E \left[ S_t \right] \theta_j, \quad (16)$$

$$VAR \left[ Y_t^j \right] = VAR \left[ S_t \right] \left( \theta_j^2 + \frac{\sigma_j^2 \lambda}{1 - \alpha} \right). \quad (17)$$
Properties

By Kim et al. 2012, if $w \in \mathbb{R}^n$, then the random variable $X = w' Y_t$ has characteristic function (15) with parameters $(a, \lambda, C, \tilde{\theta}, \tilde{\mu}, \tilde{\sigma})$, where

$$\tilde{\theta} = w' \theta \quad \tilde{\mu} = w' \mu \quad \tilde{\sigma} = \sqrt{w' \Sigma w}.$$ 

This result will be useful to compute portfolio risk measures.
Having defined a MNTS random variable, we assume that the price at time $t$ of the stock $j$ is given by the following equation

$$P_t^j = P_{t-1}^j \exp(Y_t^j),$$

(18)

where $P_t^j$ and $P_{t-1}^j$ are the price of the stock $j$ at times $t$ and $t - 1$, respectively, and $Y_t^j$ is the log-return $r_{j,t}$ of the $j$-th underlying asset over the interval $[t - 1, t]$, for every $j = 1, \ldots, n$, that is

$$r_{j,t} = \log \left( \frac{P_t^j}{P_{t-1}^j} \right).$$

We refer to the model in equation (18) to as the continuous-time MNTS model.
The discrete-time model

For each stock \( j \) from 1 to \( n \), we consider the asymmetric Glosten-Jagannathan-Runkle (GJR) model (Glosten et al., 1993)

\[
 r_{j,t} = m_j + \eta_{j,t} \tag{19}
\]

where \( \eta_{j,t} = \sigma_{j,t} \varepsilon_{j,t} \)

\[
 \sigma_{j,t}^2 = \alpha_{0,j} + \alpha_{1,j} \eta_{j,t-1}^2 + \omega_j l_{t-1} \eta_{j,t-1}^2 + \beta_{1,j} \sigma_{j,t-1}^2 \tag{20}
\]

and \( l_{t-1} = 1 \) for negative residuals, otherwise it is zero. The set of constant parameters is \((\alpha_{0,j}, \alpha_{1,j}, \beta_{1,j}, \omega_j)\). If \( \omega_j > 0 \), then the model considers the leverage effect, that is, bad news raises the future volatility more than good news.

In practice

1. estimation of the model’s parameters by assuming that \( \eta_{j,t} \) are normally distributed
2. estimation of the filtered (or devolatized) data by taking \( \varepsilon_t = \Delta_t^{-1} (r_t - m) \) where \( \Delta_t \in \mathbb{R}^{n \times n} \) is the diagonal matrix with \( j \)-th diagonal element \( \sigma_{j,t} \)
3. fitting of the MNTS distribution on the random vector \( \varepsilon \)
The density function

Note that the density function of a MNTS distribution can be written as

\[ f_Y(y; \Theta) = \int_0^\infty f_{Y|S}(y|s; \mu, \theta, \Sigma) h(s; a, \lambda, C) ds \]  \hspace{1cm} (21)

where \( Y|S \sim N(\mu + \theta S, S\Sigma) \), \( h \) is the density function of the subordinator defined in equation (6), and \( \Theta \) is the set of model parameters \((a, \lambda, C, \theta, \mu, \Sigma)\).
Given a set of $N$ observations $\{Y^k = Y_{tk} - Y_{tk-1}\}_{k=1,...,N}$, the log-likelihood related to the model (10) can be written as

$$LL(\Theta; Y^1, \ldots, Y^N) = \sum_{k=1}^{N} \log f_Y(Y^k; \Theta).$$  \hspace{1cm} (22)

We consider the following likelihood function instead of the likelihood in equation (22)

$$LL(\Theta; Y^1, \ldots, Y^N, S^1, \ldots, S^N) = \sum_{k=1}^{N} \log f_{Y,S}(Y^k, S^k; \Theta)$$

$$= \sum_{k=1}^{N} \log f_{Y|S}(Y^k|S^k; \mu, \theta, \Sigma) + \sum_{k=1}^{N} \log h_S(S^k; a, \lambda, C)$$

$$= L_1(\mu, \theta, \Sigma; Y|S) + L_2(a, \lambda, C; S)$$ \hspace{1cm} (23)

where $\Theta = \{a, \lambda, C, \theta, \mu, \Sigma\}$ is the set of parameters, $\{S^k = S_{tk} - S_{tk-1}\}_{k=1,...,N}$ the latent mixing variables coming from the representation (10). In order to find a maximum likelihood estimation based on (23), we consider the following iterative algorithm.
The algorithm

The *expectation-conditional maximization either* (ECME) algorithm:

1. Set \( i = 1 \) and select a starting value for \( \Theta^{(1)} \), that is \( \mu^{(1)} \in \mathbb{R}^n \) is the sample mean, \( \theta^{(1)} \in \mathbb{R}^n \) is the zero vector, \( V \in \mathbb{R}^n \times \mathbb{R}^n \) is the sample covariance matrix.

2. By considering that

\[
f_{S|Y^k}(s; Y^k, \Theta) = \frac{f_{Y|S}(Y^k|s; \mu, \theta, \Sigma)h(s; a, \lambda)}{f_Y(Y^k; \Theta)} \tag{24}
\]

compute the following weights

\[
\delta^{(\cdot)}_k = \mathbb{E}(S^k^{-1}|Y^k, \Theta^{(\cdot)}), \quad \eta^{(\cdot)}_k = \mathbb{E}(S^k|Y^k, \Theta^{(\cdot)}), \tag{25}
\]

3. Evaluate the average values

\[
\bar{\delta}^{(i)} = \sum_{k=1}^{N} \delta^{(i)}_k, \quad \bar{\eta}^{(i)} = \sum_{k=1}^{N} \eta^{(i)}_k.
\]
4. Get the estimates

\[
\theta^{(i+1)} = \frac{N^{-1} \sum_{k=1}^{N} \delta_k^{(i)} (\bar{Y} - Y^k)}{\delta(i) \bar{\eta}^{(i)} - 1},
\]

\[
\mu^{(i+1)} = \frac{N^{-1} \sum_{k=1}^{N} \delta_k^{(i)} Y^k - \theta^{(i+1)}}{\delta(i)},
\]

\[
\Psi = \frac{1}{N} \sum_{k=1}^{N} \delta_k^{(i)} (Y^k - \mu^{(i+1)}) (Y^k - \mu^{(i+1)})' - \bar{\eta}^{(i)} \theta^{(i+1)} \theta^{(i+1)'},
\]

\[
\Sigma^{(i+1)} = \frac{|V|^{1/n} \Psi}{|\Psi|^{1/n}}.
\]

5. Set

\[
\Theta^{(i')} = \{a^{(i)}, \lambda^{(i)}, C^{(i)}, \theta^{(i+1)}, \mu^{(i+1)}, \Sigma^{(i+1)}\}
\]

and calculate the new weight \(\bar{\eta}^{(i')}\) as done in Steps 2 and 3.
6. To complete the calculation of $\Theta^{(i+1)}$, find $a$, $\lambda$, and $C$ that maximize the likelihood function in equation (22), that is

$$LL(\Theta^{(i+1)}; Y^1, \ldots, Y^N) = \sum_{k=1}^{N} \log f_Y(Y^k; \Theta^{(i+1)})$$ (26)

where $\Theta^{(i+1)} = \{a, \lambda, C, \theta^{(i+1)}, \mu^{(i+1)}, \Sigma^{(i+1)}\}$.

7. If $i < 1,000$ and $LL(i) - LL(i - 1) > 1e - 5$, increment iteration count $i$ and go to step 2, otherwise, stop the algorithm.
Data

We obtained from Bloomberg daily dividend adjusted closing prices from January 2, 1990 through December 31, 2012 for five selected companies included in the S&P 500: Apple Inc. (ticker APPL), Dell Inc. (ticker DELL), International Business Machines Corp. (ticker IBM), Hewlett-Packard Comp. (ticker HPQ), Microsoft Corp. (ticker MSFT).

For each trading day from January 2, 2008 to December 31, 2012, a window of fixed size is considered (1,500 trading days) for a total of 1,259 rolling windows estimations (for the 1\textsuperscript{st} trading day January 2, 2008, the time-series end on January 2, 2008, ...)

We analyze

- 3 continuous time models (normal, MGH and MNTS)
- 3 discrete-time models (normal-, MGH-, and MNTS-Garch)
Simulations

- Historical data
- Simulated normal
- Simulated MGH
- Simulated MNTS
Some result

- Test estimates
- MNTS parameters
- MNTS Garch parameters
- MGH parameters
- MGH Garch parameters
- Test simulation / estimation
Definitions

Recall that the value at risk (VaR) at tail probability level $\delta$ is defined as

$$VaR_\delta(X) = -\inf\{x \mid P(X \leq x) > \delta\} = -F_X^{-1}(\delta)$$

and can be computed by inverting the cumulative distribution function $F_X$, and that the AVaR of a continuous random variable $X$ with finite mean (i.e. $E[X] < \infty$) at tail probability level $\delta$ is defined as the average of the VaRs that are greater than the VaR at tail probability $\delta$, that is

$$AVaR_\delta(X) = \frac{1}{\delta} \int_0^\delta VaR_p(X) dp = -E[X \mid X < -VaR_\delta(X)].$$

Therefore, by construction, AVaR is focused on the losses in the tail that are greater than the corresponding VaR level. From Kim et al. (2010-2011) it is possible to obtain a closed formula (up to an integration) to compute the average value at risk (AVaR) when only the characteristic function is known in closed form.
Backtesting

We consider

1. the LR test of unconditional coverage ($LR_{uc}$), which is the same as the proportion of failures test by Kupiec (1995)
2. the LR test of independence ($LR_{ind}$)
3. the joint test of coverage and independence ($LR_{cc}$)

as described in Christoffersen (1998 and 2010).
A risk assessment study

\[ \text{VaR (a)} \]

\[ \text{normal} \]

\[ \text{MGH} \]

\[ \text{MNTS} \]

\[ \text{Jan09 Jan10 Jan11 Jan12} \]

\[ \text{VaR (b)} \]

\[ \text{normal Garch} \]

\[ \text{MGH Garch} \]

\[ \text{MNTS Garch} \]

\[ \text{Jan09 Jan10 Jan11 Jan12} \]

\[ \text{AVaR (a)} \]

\[ \text{normal} \]

\[ \text{MGH} \]

\[ \text{MNTS} \]

\[ \text{Jan09 Jan10 Jan11 Jan12} \]

\[ \text{AVaR (b)} \]

\[ \text{normal Garch} \]

\[ \text{MGH Garch} \]

\[ \text{MNTS Garch} \]

\[ \text{Jan09 Jan10 Jan11 Jan12} \]
Conclusions and further research

Conclusions:
- we are able to ride with the four horsemen
- we proposed an EM-based algorithm that can be extended to a broader class of models
- the model is computationally tractable

Further research:
- empirical analysis on the major European stock market indexes and/or a larger number of stocks
- option pricing (double-estimation: historical and risk-neutral) and hedging
- extend the model to other asset classes (i.e. interest rate derivatives)
- find a faster historical estimation procedure
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