Dynamic Portfolio Management with Views at Multiple Horizons

Attilio Meucci and Marco Nicolosi

Discussant: Dario Alitab

XVI Workshop on Quantitative Finance
Parma, 30 January 2015
Subject: Quantitatively including subjective views in a base-case risk portfolio investment strategy over multiple time horizons;

Previous one-period approaches:
- Black and Litterman (1990): Normal distributed risk factors, views as expectations of linear combination of the risk factors;

Novelty proposal: Dynamic Entropy Pooling to optimally determine the exposures to the risk factors reflecting the discretionary views on a multi-period allocation process.
Dynamic Entropy Pooling

- Identify a set of risk drivers $X_{t \rightarrow t}$ for the assets under management for a multi-period portfolio;
- Choose a model for the dynamics of the risk drivers and estimate it ("prior" distribution $\bar{f}(X_{t \rightarrow t})$);
- State your views on generic functions of the risk drivers $\mathcal{V}_t : g(X_{t \rightarrow t})$;
- Determine the "posterior" distribution $\bar{f}(X_{t \rightarrow t})$ in accordance to $\mathcal{V}_t$ via relative entropy minimization

$$
\mathcal{E}(f, \bar{f}) \equiv \int f(X_{t \rightarrow t}) \ln \frac{f(X_{t \rightarrow t})}{\bar{f}(X_{t \rightarrow t})} dX_{t \rightarrow t}
$$

$$
\bar{f}(X_{t \rightarrow t}) \equiv \arg \min_{f \in \mathcal{V}} \{ \mathcal{E}(f, \bar{f}(X_{t \rightarrow t})) \}
$$

$\mathcal{V}$: the set of distribution consistent with the views statements.
Dynamic Entropy Pooling

- Define the portfolio profit and loss with vector of exposures $b_{t\rightarrow\bar{t}}$ whose generic unit period component is $\Pi_{(s,s+1]} = p&l (X_{s+1}, X_s, \ldots, X_{t+1}; b_s, \ldots, b_t)$ for $s \geq t$;

- Define for each time an index of satisfaction $S_t$ for the multi-period investment as a function of the future exposures and distribution of the risk drivers $S_t\{\Pi_{t\rightarrow\bar{t}}\} = S_t (b_{t\rightarrow\bar{t}}, \bar{f} (X_{t\rightarrow\bar{t}}))$;

- Construct your portfolio at any time $t$ optimizing the index of satisfaction as a function of future exposures $b_t^* \equiv \arg \max_{b_t \in C_t} \{S_t (b_t, \bar{f} (X_{t\rightarrow\bar{t}}))\}$;

- Proposed expression for the index of satisfaction

$$S_t = \sum_{s=t}^{\bar{t}} \left[ \mathbb{E}_t \{\Pi_{(s,s+1]}\} - \frac{\gamma}{2} \text{Var}_t \{\Pi_{(s,s+1]}\} - \frac{\eta}{2} \mathbb{E}_t \{\text{MI}_s\} \right]$$
Questions and Comments

- What are the difficulties to extend Black Litterman to a multi-period portfolio management context which are overcome by Dynamic Entropy Pooling approach?
Questions and Comments

- Employing DEP for an investment model accounting for a sub-linear market impact term as shown in different studies for example Lillo et al. (2008) and Toth et al. (2011).

\[ \Delta(Q) = Y \sigma \sqrt{\frac{Q}{V}} \]

\( Y \approx 1 \)

\( \sigma \): daily volatility;

\( Q \): size of a metaorder;

\( V \): daily traded volume.
Questions and Comments

- Detail an application for financial institutes interested in determining their optimal allocation in accordance to capital requirements imposed by Regulators and assessing, via stress-testing, their robusteness in an economic crisis.