Dynamic Portfolio Management with Views at Multiple Horizons

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The usual approach to discretionary portfolio management is to use subjective views that refer to the distribution of the market at one specific investment horizon.

However, portfolio managers operate with multiple horizons.

We provide a quantitative approach to perform dynamic portfolio management with discretionary, non-synchronous views.

The model allows to consider views at multiple horizons, including calendar statements such as expectation on growth of a factor.

The model allows to process views on external factors such as macroeconomic variables.
Outline

- Dynamic of the market risk drivers
- Prior model + views $\rightarrow$ Posterior model
- From the risk drivers to the portfolio profit and loss (assumption of linearity)
- Myopic one-period mean-variance case
- Allocation with market impact of transaction
- Two case studies
The Ornstein-Uhlenbeck process

Consider a book of assets driven by $\bar{n}$ risk drivers: $X_t$
We assume that the drivers follow a MVOU process:

$$dX_t = (-\theta X_t + m) dt + \sigma dW_t$$

Choose a set of discrete monitoring dates $t, t + 1, \ldots, \bar{t}$
Stack the process at the monitoring times as follows:

$$X_{t+1 \rightarrow \bar{t}} \equiv \begin{pmatrix} X_{t+1} \\ \vdots \\ \dot{X}_{\bar{t}} \end{pmatrix}$$

then

$$X_{t+1 \rightarrow \bar{t}}|i_t \sim \mathcal{N} \left( \mu_{t+1 \rightarrow \bar{t}}, \sigma^2_{t+1 \rightarrow \bar{t}} \right)$$
The expectation is
\[
\mu_{t+1 \rightarrow \bar{t}} \equiv \begin{pmatrix}
    e^{-\theta^2} x_t + (\mathbb{I}_n - e^{-\theta^2}) \theta^{-1} m \\
    \vdots \\
    e^{-(\bar{t}-t)\theta^2} x_t + (\mathbb{I}_n - e^{-(\bar{t}-t)\theta^2}) \theta^{-1} m
\end{pmatrix}
\]

The covariance matrix is
\[
\sigma_{t+1 \rightarrow \bar{t}}^2 \equiv \begin{pmatrix}
    \sigma_1^2 & \frac{\sigma_1^2}{\theta} e^{-\theta^2} & \cdots & \frac{\sigma_1^2}{\theta} e^{-(\bar{t}-t-1)\theta^2} \\
    \frac{\sigma_1^2}{\theta} & \sigma_2^2 & \cdots & \frac{\sigma_2^2}{\theta} e^{-(\bar{t}-t-2)\theta^2} \\
    \vdots & \vdots & \ddots \\
    \frac{\sigma_1^2}{\theta} & \frac{\sigma_2^2}{\theta} & \cdots & \sigma_{\bar{t}-t}^2
\end{pmatrix}
\]

where
\[
\text{vec}(\sigma_{t+1 \rightarrow \bar{t}}^2) \equiv (\theta \oplus \theta)^{-1} (\mathbb{I}_n^2 - e^{-(\theta \oplus \theta)^2}) \text{vec}(\sigma^2)
\]
We extend the entropy pooling approach in Meucci (2010) to the case of multiple horizons.

**The prior:** assume a model for the joint distribution of the process at the monitoring times \( \mathbf{X} \sim f(\mathbf{X}_t, \ldots, \mathbf{X}_{\tilde{t}}) \)

**The views:** are a set of statements \( \mathbb{V} \) on arbitrary functions of \( \mathbf{X} \): \( \mathbb{V} = g(\mathbf{X}) \).

**The posterior:** is defined as the closest distribution to the prior that satisfies the views:

\[
\bar{f} \equiv \arg\min_{f \in \mathbb{V}} \{ \mathcal{E}(f, \bar{f}) \}
\]

where the relative entropy is defined as:

\[
\mathcal{E}(f, \bar{f}) \equiv \int f(\mathbf{x}_t, \ldots, \mathbf{x}_{\tilde{t}}) \ln \frac{f(\mathbf{x}_t, \ldots, \mathbf{x}_{\tilde{t}})}{\bar{f}(\mathbf{x}_t, \ldots, \mathbf{x}_{\tilde{t}})} d\mathbf{x}_t \cdots d\mathbf{x}_{\tilde{t}},
\]
For analytical tractability, we consider views of the form:

\[ V_t : \begin{align*}
\mathbb{E}\{v_\mu X_{t \rightarrow \bar{t}} | i_t\} &\equiv \mu_{\text{view}} \\
\mathbb{C}_V\{v_\sigma X_{t \rightarrow \bar{t}} | i_t\} &\equiv \sigma^2_{\text{view}}.
\end{align*} \]

- \( v_\mu \) and \( v_\sigma \) are matrices that define arbitrary linear combinations of the process at the times for the views.
- For a MVOU process the prior is joint normal. Then, the full-confidence posterior is

\[ X_{t+1, \ldots, \bar{t}} | i_t \sim N \left( \mu_{t+1 \rightarrow \bar{t}}, \sigma^2_{t+1, \ldots, \bar{t}} \right) \]
The posterior expectation

- We define the pseudo inverse matrix of $\mathbf{v}_\mu$:

$$
\mathbf{v}_\mu^+ \equiv \sigma_{t+1 \rightarrow \bar{t}}^2 \mathbf{v}_\mu \left( \mathbf{v}_\mu \sigma_{t+1 \rightarrow \bar{t}}^2 \mathbf{v}_\mu \right)^{-1}
$$

- We also define the two complementary projectors:

$$
\mathbb{P}_\mu \equiv \mathbb{I}_{\bar{n}(\bar{t} - t)} - \mathbf{v}_\mu^+ \mathbf{v}_\mu \quad \text{and} \quad \mathbb{P}_\mu^\perp \equiv \mathbf{v}_\mu^+ \mathbf{v}_\mu
$$

- Then

$$
\overline{\mu}_{t+1 \rightarrow \bar{t}} \equiv \mathbb{P}_\mu \mu_{t+1 \rightarrow \bar{t}} + \mathbb{P}_\mu^\perp (\mathbf{v}_\mu^+ \mu_{view})
$$
The structure is the same as for the posterior expectation. We define the pseudo inverse matrix of $\mathbf{v}_\sigma$:

$$\mathbf{v}_\sigma^+ \equiv \sigma_{t+1 \rightarrow \bar{t}}^2 \mathbf{v}'(\mathbf{v}_\sigma \sigma^2_{t+1 \rightarrow \bar{t}} \mathbf{v}')^{-1}$$

and the two complementary projectors:

$$\mathbb{P}_\sigma \equiv \mathbb{I}_{\bar{n}(\bar{t}-t)} - \mathbf{v}_\sigma^+ \mathbf{v}_\sigma \quad \text{and} \quad \mathbb{P}_\perp \equiv \mathbf{v}_\sigma^+ \mathbf{v}_\sigma$$

Then

$$\sigma_{t+1 \rightarrow \bar{t}}^2 \equiv \mathbb{P}_\sigma \sigma^2_{t+1 \rightarrow \bar{t}} \mathbb{P}' + \mathbb{P}_\perp \mathbf{v}_\sigma^+ \sigma^2_{\text{view}} (\mathbf{v}_\sigma^+)'(\mathbb{P}_\perp)'$$
The Profit and Loss (P&L)

- We assume linearity of the P&L in the increments of the risk drivers $X$ through a set of money exposures $b$:

$$\Pi_{(t,t+1]} = b'_t (X_{t+1} - X_t)$$

- The risk drivers follow a MVOU process filtered by views at multiple horizons.

- The set of risk drivers can be extended to include also external factors that do not affect directly the P&L of the instruments.

- On such additional factors we can express views that therefore influence the P&L through correlation.

- The corresponding entries in the exposures vector $b$ will be set to zero.
The P&L: equity

Consider an equity share or an index. Then the risk driver is its log-value: $X_t = \ln V_t$.

The P&L of a portfolio with $h_{n,t}$ shares in the $n$-th asset is:

$$\Pi_{(t,t+1]} = \sum_n h_{n,t} V_{n,t} \times \left( \frac{V_{n,t+1}}{V_{n,t}} - 1 \right) \approx \sum_n b_{n,t} \Delta X_{n,t+1}$$

More in general, in terms of a style/risk linear factor model:

$$\Pi_{(t,t+1]} = \sum_k b_{k,t}^{\text{style}} \Delta X_{k,t+1}^{\text{style}}$$
Suppose that the $n$-th asset is a fixed income instrument. Its value at the first order satisfies

$$\Pi_{n,(t,t+1]} \approx -\sum_k dv01_{n,k,t} \Delta X_{k,t+1}$$

- $X_{k,t}$ is the $k$-th key-rate on the par yield curve
- $dv01_{n,k,t}$ is the dollar-sensitivity of the $n$-th instrument to $X_{k,t}$

Then the P&L due to a set of fixed income instruments is:

$$\Pi_{(t,t+1]} \approx \sum_k \left( -\sum_n h_{n,t} dv01_{n,k,t} \right) \Delta X_{k,t+1}$$
The P&L: stock options

- For a stock option, the risk drivers are the log-value of the underlying \( X = \ln V \) and the implied volatility \( \Sigma^{impl} \).

- Then for a portfolio of stock options, the P&L is:

\[
\Pi_{(t,t+1]} \approx \sum_n h_{n,t} \delta_{n,t} V_{n,t} \Delta X_{n,t+1} + \sum_n h_{n,t} \nu_{n,t} \Delta \Sigma_{n,t+1}^{impl}
\]

where \( \delta_{n,t} \) and \( \nu_{n,t} \) are the delta and vega of the \( n \)-th option.

- In large portfolio of stock options, the underlyings are summarized by a style/risk factor model and the VIX index is used for the implied volatility.

\[
\Pi_{(t,t+1]} \approx \sum_k b_{k,t}^{style} \Delta X_{k,t+1}^{style} + b_t^{VIX} \Delta VIX_{t+1}
\]
In absence of market impact, the optimal exposure at each trading time $t \in \{0, 1, 2 \ldots \}$ is obtained by maximizing the expected P&L penalized by risk aversion.

The prior solution is:

$$b^\text{Prior}_t = \frac{1}{\gamma} \times \frac{1}{(\mathbb{C}_\mathbb{V}\{\Delta X_{t+1} | i_t \})^{-1}} \times \left( \mathbb{E}\{\Delta X_{t+1} | i_t \} \right)^{-1} \times \left( \prod_{i}^n - e^{-\theta} \right) \left( \theta^{-1} \mathbf{m} - \mathbf{x}_t \right)$$

The solution is driven by the dislocation between the current time’s values of the risk drivers $x_t$ and $\theta^{-1} \mathbf{m}$ which is the vector of the long term expected levels.
Myopic mean-variance trade-off: posterior solution

- We have to consider the posterior vector of the expected returns and the covariance matrix of the joint process $\bar{f}(x_t, \ldots x_{\bar{t}})$
- Then we select the components relative to the next trading period, using a matrix selector $\rho$: $\Delta \bar{\mu}_{t+1} \equiv \rho \Delta \bar{\mu}_{t+1 \rightarrow \bar{t}}$
- The posterior solution is:

$$b^*_t = \frac{1}{\gamma} (\bar{\sigma}_{1}^2)^{-1} \rho \mathbb{P}_\mu \left( \begin{pmatrix} \mathbb{I}_{\bar{n}} - e^{-1\theta} \\ \vdots \\ \mathbb{I}_{\bar{n}} - e^{-(\bar{t} - t)\theta} \end{pmatrix} \right) (\theta^{-1} m - x_t)$$

$$= \frac{1}{\gamma} (\bar{\sigma}_{1}^2)^{-1} \rho \mathbb{P}_\mu (x_t)$$

$$+ \frac{1}{\gamma} (\bar{\sigma}_{1}^2)^{-1} \rho \mathbb{P}_\mu^\perp \left( v^+_\mu \mu_{\text{view}} - \begin{pmatrix} x_t \\ \vdots \\ x_t \end{pmatrix} \right)$$

$$= b_t^{\text{LongTerm}} + b_t^{\text{ViewMean}}$$
Market impact

- Consider trading in presence of market impact
- Empirical observations show that the impact of transaction on the P&L is a superlinear function of the trade size (see Almgren et al. 2005):
- However we assume quadratic market impact as in Garleanu and Pedersen (2013)

\[ MI_t = a^2 + \Delta b_t' c^2 \Delta b_t \]

- \( c^2 \) is a symmetric positive definite matrix. Think of it as a multidimensional version of the Kyle’s lambda.
- \( a^2 \) is the average cost of maintaining constant exposures
The satisfaction functional at time $t$ is:

$$
\overline{S}_t^{(\gamma, \eta)} \equiv \sum_{s=t}^{t+\overline{\tau}} e^{-\lambda(s-t)} \left[ \mathbb{E}\{\Pi_{(s,s+1)}|i_t\} - \frac{\gamma}{2} \mathbb{V}\{\Pi_{(s,s+1)}|i_t\} - \frac{\eta}{2} \mathbb{E}\{\Pi_{(s,s+1)}|i_t\} \right]
$$

As in Gârleanu and Pedersen (2013) this is the objective function of a manager who is compensated based on his performance on each trading period.

- $e^{-\lambda t}$ is a discount factor.
- $t + \overline{\tau}$ is the effective rolling horizon, i.e. such that the last term in the sum is negligible.
- The expectation is conditioned at information at time $t$ and it is computed under the posterior distribution.
The time line of the entropy pooling

- Start
- View time
- Generic trading time
- View time
- Effective end of risk drivers process
- Portfolio manager effective rolling horizon
- Trading time
- View time
- Effective end of horizon
A linear policy function

- The objective function depends on the exposures $B_s$, $s \geq t$, that at time $t$ are random variables.
- There are two different approaches for maximizing the objective function:
  1. solving the Bellamn equation as in Gârleanu, and Pedersen (2013)
  2. using a given functional form for the policy, as in Molleami, and M. Saglam (2012) or in Brandt et al. (2009)
- Similar to Brandt et al. (2009) we consider a linear policy function:
  $$B_s = z_s \left( \frac{1}{\chi_s} \right), \text{ for any } s = t, \ldots, t + \bar{t}$$

where $z_s$ are deterministic matrices
The objective function can be expressed in terms of the policy coefficients:

\[ \mathbf{z}_{t \to t+\bar{\tau}} \equiv \left( \begin{array}{c} \text{vec}(\mathbf{z}_t) \\ \text{vec}(\mathbf{z}_{t+1}) \\ \vdots \\ \text{vec}(\mathbf{z}_{t+\bar{\tau}}) \end{array} \right) \]

The resulting problem is quadratic:

\[ \mathbf{z}_{t \to t+\bar{\tau}}^* = \arg\min_{\mathbf{z}_{t \to t+\bar{\tau}}} \left\{ \mathbf{z}'_{t \to t+\bar{\tau}} \bar{\mathbf{q}} \mathbf{z}_{t \to t+\bar{\tau}} - \mathbf{z}'_{t \to t+\bar{\tau}} \mathbf{l} \right\} \]

As the linear assumption gives rise to slightly suboptimal exposures we re-calibrate the optimized path at each step, only keeping and implementing the first step at each iteration

\[ t \to \mathbf{b}_t^* \equiv \mathbf{z}_t^* \left( \begin{array}{c} 1 \\ \mathbf{x}_t \end{array} \right) \]
How to impose constraints such as liquidation or diversification constraints and short sale restrictions?

1. Equality constraints at time $s$

$$
\chi_s B_s = \xi_s \quad \rightarrow \quad \chi_s z_s = (\xi_s, 0)
$$

2. Probabilistic inequality constraints (see Molleami and Saglam 2012):

$$
\chi_s B_s \leq \xi_s \quad \rightarrow \quad P(\chi_s B_s > \xi_s) \leq \varepsilon
$$

That is

$$
\mathcal{N}^{-1}(1 - \varepsilon) \sqrt{\chi_s z_s \hat{\sigma}_s^2 z_s' \chi_s' \chi_s} \leq -\chi_s z_s \hat{\mu}_s + \xi_s
$$

where

$$
\hat{\mu}_s \equiv \left( \frac{1}{\mu_s} \right) \quad \text{and} \quad \hat{\sigma}_s^2 \equiv \left( \begin{array}{cc} 0 & 0' \\ 0 & \sigma_s^2 \end{array} \right)
$$
A second order cone program (SOCP)

- In the presence of linear equality and inequality constraints, and for a linear policy function, the optimization problem becomes an instance of a SOCP

\[
\mathbf{z}_{t \to t+\bar{\tau}}^* = \arg\min_{\mathbf{z}_{t \to t+\bar{\tau}}} \left\{ \mathbf{z}'_{t \to t+\bar{\tau}} \mathbf{q}_{t \to t+\bar{\tau}} - \mathbf{z}'_{t \to t+\bar{\tau}} \mathbf{l} \right\}
\]

such that

\[
\begin{cases}
\mathbf{m}_{t \to t+\bar{\tau}} \mathbf{z}_{t \to t+\bar{\tau}} = \mathbf{u} \\
\|\mathbf{m}_{j,s} \mathbf{z}_{t \to t+\bar{\tau}}\|_2 \leq \mathbf{u}'_{j,s} \mathbf{z}_{t \to t+\bar{\tau}} + \xi_{j,s} \quad s = t, \ldots, t+\bar{\tau}
\end{cases}
\]

- It can be solved numerically by using the Matlab package CVX.
The risk driver is the 10y government rate. At time $t = 0$ the rate is $x_0 = 2.61\%$

The view is that the expected value of the rate will be $\mu_{\text{view}} = x_0 - 0.5\%$ at $t^* = 1$ year

Trading is daily

The OU calibrated parameters are (on a daily basis):

$m = 0.03 \times 10^{-3}$, $\theta = 1.25 \times 10^{-3}$, $\sigma^2 = 0.23 \times 10^{-6}$

The market impact matrix is $c^2 = \sigma_1^2$

$\gamma = 10^{-2}$; $\eta = 0.5$; $\lambda$ is chosen in such a way that the half life of the discount factor is 20 days; $\bar{\tau} = 128$ days
The solution in absence of market impact

- When market impact is neglected, the prior solution is
  \[ b_t^{\text{Prior}} = \frac{2\theta}{\gamma \sigma^2} \frac{1}{1+e^{-\theta}} \left( \frac{m}{\theta} - x_t \right) \]

- The posterior solution reads:
  \[ b_t^* = \frac{2\theta}{\gamma \sigma^2} \frac{1}{1+e^{-\theta}} \left( \frac{1+e^\theta}{1+e^{\theta(t^*-t)}} \right) \left( \frac{m}{\theta} - x_t \right) + \frac{2\theta}{\gamma \sigma^2} \frac{e^\theta}{e^{\theta(t^*-t)} - e^{-\theta(t^*-t)}} \left( \mu_{\text{view}} - x_t \right) \]

\[ b_t^{\text{LongTerm}} \]
\[ b_t^{\text{ViewMean}} \]

\[ \theta \approx 0 \Rightarrow b_t^* \approx b_t^{\text{ViewMean}} \approx \frac{1}{\gamma \sigma^2 (t^*-t)} \left( \mu_{\text{view}} - x_t \right) \]

\[ t^* \rightarrow \infty \Rightarrow b_t^* \approx b_t^{\text{Prior}} \]
The market model
Dynamic portfolio management
Case studies

\( b_t^{Prior} \)

\( b_t^{LongTerm} \)

\( b_t^{viewMean} \)

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Dynamic Portfolio Management with Views at Multiple Horizons
Two risk drivers (investable/non-investable), two views

- We add to the previous case study an external factor, the 5 years TIP spread, whose value at time $t = 0$ is $x_{2,0} = 1.93\%$.
- We state a new view on the TIP spread in addition to the view on the rate: the expected value of the TIP spread will be $\mu_{2,\text{view}} = x_{2,0} + 0.5\%$ at $t^{**} = 0.75$ years.
- The calibrated model parameters are (on a daily basis):

\[
\begin{align*}
m &= 10^{-3} \begin{pmatrix} 0.03 \\ 0.19 \end{pmatrix} \\
\theta &= 10^{-3} \begin{pmatrix} 1.25 & 0 \\ 0 & 8.97 \end{pmatrix} \\
s^2 &= 10^{-6} \begin{pmatrix} 0.23 & 0.06 \\ 0.06 & 0.13 \end{pmatrix}
\end{align*}
\]
The solution in absence of market impact

\[ b_{1,t}^* = b_{1,t}^{\text{Long Term},X_1} + b_{1,t}^{\text{Long Term},X_2} + b_{1,t}^{\text{ViewMean},X_1} + b_{1,t}^{\text{ViewMean},X_2} \]

where

\[ b_{1,t}^{\text{Long Term},X_1} = \frac{2\theta_1}{\gamma\sigma_1^2} \frac{1}{1+e^{-\theta_1}} \left( 1 - \alpha_t \frac{1-e^{-\theta_1(t^*-t)}}{1-e^{-\theta_1}} \right) \left( \frac{m_1}{\theta_1} - X_1,t \right) \]

\[ b_{1,t}^{\text{Long Term},X_2} = -\beta_t \frac{2\theta_1}{\gamma\sigma_1^2} \frac{1}{1-e^{-2\theta_1}} \left( 1 - e^{-\theta_2(t^{**}-t)} \right) \left( \frac{m_2}{\theta_2} - X_2,t \right) \]

\[ b_{1,t}^{\text{ViewMean},X_1} = \alpha_t \frac{2\theta_1}{\gamma\sigma_1^2} \frac{1}{1-e^{-2\theta_1}} \left( \mu_{1,\text{view}} - X_1,t \right) \]

\[ b_{1,t}^{\text{ViewMean},X_2} = \beta_t \frac{2\theta_1}{\gamma\sigma_1^2} \frac{1}{1-e^{-2\theta_1}} \left( \mu_{2,\text{view}} - X_2,t \right) \]

and \( \alpha_t \) and \( \beta_t \) are two coefficients depending on the model parameters as well as the times of the views.
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Dynamic Portfolio Management with Views at Multiple Horizons
We presented the dynamic entropy pooling, a quantitative approach to discretionary portfolio management. This methodology allows to process views and stress testing at multiple horizons. To preserve analytical tractability, we assumed a MVOU process as the "prior" model of the risk drivers. The posterior market distribution can then be used in multi-period strategies as well as in a standard one-period framework. Among the risk drivers the manager can include also some external risk drivers, such as macroeconomic variables.
Related literature