Markovian Nash equilibrium in Financial markets with asymmetric information and related forward-backward systems

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Brief Summary

In the framework of the paper by Back, the authors aim at removing the risk neutrality of the market makers, usually assumed in the current literature. Market observations suggest a mean reversion phenomenon, that might be accounted for by the risk adversion of market makers.

By setting a zero gain condition among market makers, they show that the price set by the market makers solves a non-standard ‘quadratic’ BSDE and the insider’s optimal strategies are solutions of a forward- backward system of partial and stochastic differential equations.

They show the existence of a Markovian solution to this forward-backward system via a fixed-point argument, providing sufficient conditions for uniqueness and Markovian nature of the solution.

Finally they explicit the strategies at equilibrium.
Main Results - Strong points

- The model explains mean reversion behaviour

- Mathematically important result: existence of solution of

\[ H_t + \frac{1}{2} H_{yy} = 0 \]

\[ dY_t = \sigma d\beta_t + \frac{\sigma^2 \rho}{2N} Y_t H_y(t, Y_t) dt \]

\[ V = H(1, Y_1) \]

implies existence of solution of \( dS_t = Z_t dB_t - \frac{\rho}{2} Y_t Z_t^2 dt \) (very non standard BSDE). Schauder’s fixed point very general method.

- easy conditions on \( h \) to have uniqueness and existence of smooth transition densities
1. Demand process is modeled as

\[ Y_t = X_t + \sigma B_t \]

\( X_t \) representing the insider's demand and \( \sigma B_t \) the liquidity traders' demand. Is it possible to imagine that the noise traders' demand might include jumps? If jump process chosen well, it should be possible to keep the Markovian setting.

I believe that the optimal demand for the insider trader would stay absolutely continuous w.r.t. \( ds \), is it so?

Would the market trader be able to distinguish where the jumps come from?
2. In this setting (market makers’ risk aversion), can the insiders knowledge be represented by the value of the underlying’s at a random time?

The random time could represent a totally or partially independent default time or a threshold time.

The PDE would change. What can be possibly kept of the technique?

The fixed point technique as used in the paper looks to be very general and flexible.