Geometric asian option pricing in general affine stochastic volatility models with jumps

F. Hubalek, M. Keller-Ressel, C. Sgarra

discussed by:
F. Cordoni, University of Trento

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Main contribution

direct contribution

▶ explicit evaluation formulae for geometric asian options, both average price and average strike options, in a general affine framework;

indirect contribution

▶ general financial framework where almost any financial model fits in;
▶ general evaluation scheme for the fair price of financial options.
evaluate the probability that a given option is exercised leads to the fair price of the same option;

fundamental to know the characteristic function of the underlying;

let $X_t$ the log-price process and $V_t$ the volatility process, the couple $(X_t, V_t)$ is an affine process if

$$
\log\mathbb{E} \left[ e^{uX_t + wV_t} \bigg| X_0, V_0 \right] = \phi(t; u, w) + V_0 \psi(t; u, w) + X_0 u;
$$

any affine process is completely determined by two functions, known as \textit{functional characteristic}

$$
F(u, w) := \frac{\partial \phi}{\partial t}(t, u, w) \bigg|_{t=0^+}, \quad R(u, w) := \frac{\partial \psi}{\partial t}(t, u, w) \bigg|_{t=0^+}.
$$
the functions $\phi$ and $\psi$ in (1) satisfy the following generalized Riccati equations

\begin{align}
\begin{cases}
\partial_t \phi &= F(u, \psi), \\
\phi(0, u, w) &= 0,
\end{cases}
\begin{cases}
\partial_t \psi &= R(u, \psi), \\
\phi(0, u, w) &= w,
\end{cases}
\end{align}

if we know $F(u, w)$ and $R(u, w)$ of the joint process $(X_t, V_t)$, then solving the Riccati equations (2) we have the functions $\phi$ and $\psi$ and thus the characteristic function of the couple $(X_t, V_t)$;

the fair price $P$ of an option is reduced to an inverse Laplace transform

\[ P = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} F(u)e^{\kappa(u; \phi, \psi)} du. \]
Main results

Exploiting previous methods they were able to retrieve closed or semi-closed formulae for $\phi$ and $\psi$ for many stochastic volatility models (possibly with jumps) such as:

- Heston model;
- Bates model;
- Turbo-Bates model;
- Barndorff-Nielson-Shepard model;
- OU time-changed Lévy process;
- CIR time-changed Lévy process.
the theory is well motivated by applications;
interesting setting that allows for good analytical results;
wide number of models fit into the present setting;
simple yet non trivial idea;
the evaluation scheme introduced allows to deal non only with european option but also with more complicated exotic/path-dependent options.
Questions

- functional characteristic;
- numerical evaluation of inverse Laplace transform;